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Investigation of seismic behavior of fluid–rectangular tank–soil/foundation systems in frequency domain

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Abstract

Main purpose of this study is to evaluate the dynamic behavior of fluid–rectangular tank–soil/foundation system with a simple and fast seismic analysis procedure. In this procedure, interaction effects are presented by Housner's two mass approximations for fluid and the cone model for soil/foundation system. This approach can determine; displacement at the height of the impulsive mass, the sloshing displacement and base forces for the soil/foundation system conditions including embedment and incompressible soil cases. Models and equations for proposed method were briefly explained for different tank–soil/foundation system combinations. By means of changing soil/foundation conditions, some comparisons are made on base forces and sloshing responses for the cases of embedment and no embedment. The results showed that the displacements and base shear forces generally decreased, with decreasing soil stiffness. However, embedment, wall flexibility, and soil–structure interaction (SSI) did not considerably affect the sloshing displacement.

Keywords: Rectangular tanks; Soil–structure interaction; Seismic behavior; Cone model; Frequency domain analysis

1. Introduction

Ground-level rectangular tanks are used to store variety of liquids, e.g. water for drinking and fire fighting, chemicals, petroleum, and liquefied gas, etc. Therefore, this type of structures must show satisfactory performance, especially, during earthquakes. Due to the inadequately designed and/or detailed tanks, significant damages have been experienced in previous incidences. Earthquake damage to tanks may occur in several forms and result in a variety of undesirable consequences such as shortage of water, spillage of dangerous chemical and liquefied gases and resultant contaminations and fires. Only the above-mentioned post-earthquake incidences may leave a more dramatic scene then the earthquake itself does [1].

Numerous studies have been carried out about seismic behavior of ground-level cylindrical tanks. However, the conditions are not the same for underground tanks, rectangular tanks, and elevated tanks. Almost the entire major studies about this subject may be summarized as follows: Hoskins and Jacobsen [2] gave the first report on analytical and experimental observations of rigid rectangular tanks under a simulated horizontal earthquake excitation. Then Graham and Rodriguez [3] used a spring-mass analogy for the fluid in a rectangular container. Housner [4,5] proposed a simple procedure for estimating the dynamic fluid effects of a rigid rectangular tank excited horizontally by an earthquake, and finally, Epstein [6] extended Housner's procedures in the sense of practical design rule.

After above studies, some researchers carried out investigations about specific topics on rectangular tanks. For instance sloshing response of the rectangular tanks was specifically investigated [7–9]. Seismically induced bending moments of the walls was examined by Haroun [10]. Flexibility of the tank walls and hydrodynamic pressure acting on the wall were studied [1,11–15]. Experimental studies to determine of the dynamic characteristic of the rectangular tanks were executed by Minowa [16] and Koh et al. [17]. Although Park et al. [18] included soil–structure interaction (SSI) to determine the hydrodynamic pressure

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acting on wall of rectangular tank in their studies; they did not propose a practical procedure considering the interaction effects. In addition, no studies were introduced on uplift behavior of the rectangular tanks. However, Priestley et al. [1] suggested a procedure that is based upon the principles for cylindrical tanks.

Previous studies concluded that, there is a significant need for a procedure that can practically cover fluid–rectangular tank–soil/foundation interaction altogether. This paper aims to investigate how the SSI effect can be practically taken into account in the seismic analysis of the rectangular tanks by considering fluid–structure interaction effects in a fast and practical manner.

2. Model for fluid–rectangular tank–soil/foundation system

Different methods and/or approaches have been employed in modeling the soil and fluid medium interacting with structures. The problem seen in Fig. 1 illustrates a complex phenomenon that includes both fluid–structure and soil/foundation interaction effects together for rectangular tanks. Being very significant, even in static cases, these phenomenons are more effective especially during the earthquakes. Therefore, these interaction effects should be considered practically for seismic analysis of the tanks. The cone model for consideration of SSI effects and Housner's two mass approximations for fluid–structure interaction are used. These approaches and the complete model proposed for the fluid–rectangular tank–soil/foundations system are explained under later in this study.

2.1. Fluid–structure interaction

Fluid–structure interaction problems can be investigated by using different approaches and methods such as: Added mass, Lagrangian, Eulerian, Lagrangian–Eulerian in the finite element method (FEM); smoothed particle hydrodynamic (SPH) methods [19]; analytical methods like Housner's two mass representations [5] and multi mass presentations of Graham and Rodriguez [3]. Among these, Housner's two mass representations is used to model fluid–tank interaction in this study. Because this approximation has been widely used in the literature and/or referred by codes.

Housner [4,5] suggested that an equivalent impulsive mass and a convective mass should represent the dynamic behavior of the fluid as an approximation of the two mass models as shown in Fig. 2. The other researchers such as Graham–Rodrigez [3] proposed methods including additional higher modes of the convective masses. According to the literature, although only the first convective mass may be considered [5], additional higher modes of convective masses may also be included in the ground-supported tanks. Generally, a single convective mass is used for

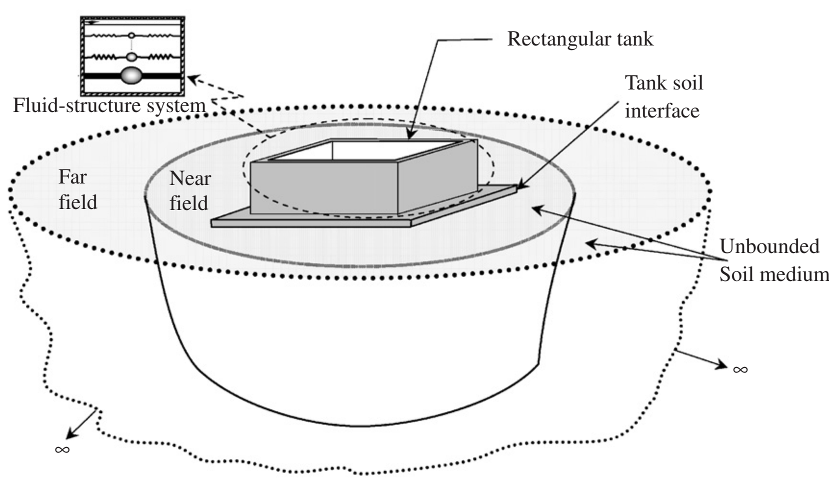

Fig. 1. The problem investigated for dynamic fluid–rectangular tank–soil interaction.

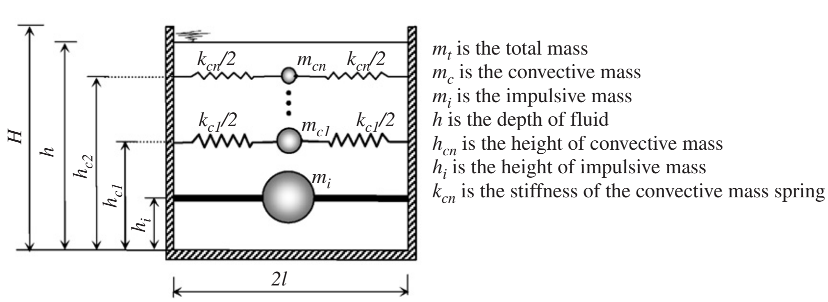

Fig. 2. The equivalent spring-mass model of the fluid.

practical design of the tanks. That approach presumes the higher modes of the sloshing have negligible influence on the forces exerted on the container even if the fundamental frequency of the structure is a close proximity one of the natural frequencies of the sloshing [20]. As in the practical analysis presented in this study, only one convective mass is considered in the numerical example. Of all expression given in Table 1, proposed ones by Housner and improved by Epstein [6] are used to be undertaken fluid characteristics.

2.2. SSI

A bounded structure consisting of the actual structure and an adjacent irregular near field soil will interact with the unbounded (infinite or semi-infinite) far field soil, assuming linear elastic model for the soil extending to infinity in the dynamic SSI analysis. This interaction can be considered in different ways. The first one is a simple methods that can be constructed by modifying the fixed base solution of the structural system [21]. This method has been widely used in the studies [22,23] and the codes such as ATC-1978, FEMA [23–25], and Eurocode 8 (EC 8) [26]. However, to represent the property of the elastic or viscoelastic half-space accurately, the stiffness and damping should be determined due to frequency of excitation. The second one is the substructure method that can consider the frequency dependent or independent dynamic stiffness and the damping of the soil/foundation system. If the frequency-dependent dynamic stiffness or the damping is required to be considered, the governing equation for the structure foundation system is expressed and solved in the frequency domain using Fourier or Laplace transformations [27–29]. The third one is the direct method that finite element and boundary element methods or a mixture of these is used in the time or frequency domain [30,31].

Table 1
Simple procedures for the fluid–rectangular tanks interaction

	Housner [4,5], Epstein (1974)		Graham-Rodrigez [3]
	$h/l \leqslant 1.5$	$h/l > 1.5$[a]	
m_{cn}/m_t	$0.527\frac{l}{h}\tanh\left(1.581\frac{h}{l}\right)$	$\frac{0.518l}{h}$	$\frac{16l}{\pi^3(2n-1)^3h}\tanh\frac{(2n-1)\pi h}{2l}$
m_i/m_t	$\frac{h}{1.732l}\tanh\left(1.732\frac{l}{h}\right)$	$\frac{1.064l}{h}$	$1-\sum_{n=1}^{\infty}\frac{m_{on}}{m_t}$
h_{cn}/h	$1-\frac{\cosh(1.581(h/l))-1}{1.581\frac{h}{l}\sinh(1.581(h/l))}$	$1-\frac{0.525l}{h}$	$1-\frac{4l}{(2n-1)\pi h}\tanh\frac{(2n-1)\pi h}{4l}$
h_i/h	$\frac{3}{8}$	$1-\frac{15l}{16h}$	$\frac{1}{2}-\frac{m_t}{m_i}\sum_{n=1}^{\infty}\frac{m_{on}}{m_t}\frac{h_{on}}{h}$
k_{cn}	$m_{on}\frac{g}{l}1.581\tanh\frac{1.581h}{l}$		$\frac{8gm_t}{h(2n-1)^2\pi^2}\tanh^2\frac{(2n-1)\pi h}{2l}$

[a] For the deep tanks ($h/l > 1.5$) Housner's proposed to consider residual mass $m_a = ((1-3l/2h)m_t)$ rigidly supported at he height of the h_a ($=1/2-3l/4h$).

The most striking feature in an unbounded soil is the radiation of energy towards infinity, leading to so-called radiation damping even in a linear system. Mathematically, in a frequency-domain analysis, the dynamic stiffness relating the amplitudes of the displacements to those of the interaction forces in the nodes of the structure–soil interface of the unbounded soil is a complex function [29]. This occurs when the unbounded soil consists of a homogeneous half-space. Hence, Wolf and Meek [32–35] proposed a cone model. Compared to more rigorous numerical methods, this cone model requires only a simple numerical manipulation within reasonable accuracy [28].

The cone model is simply summarized; for load applied directly to the structure, the soil can be represented by a static spring or the structure can even be regarded as built-in [32,34]. These static stiffnesses are expressed in the literature with a different theory. From all, the Boussinesq theory can be summarized as follows: when a homogeneous half-space is statically loaded, the variations of

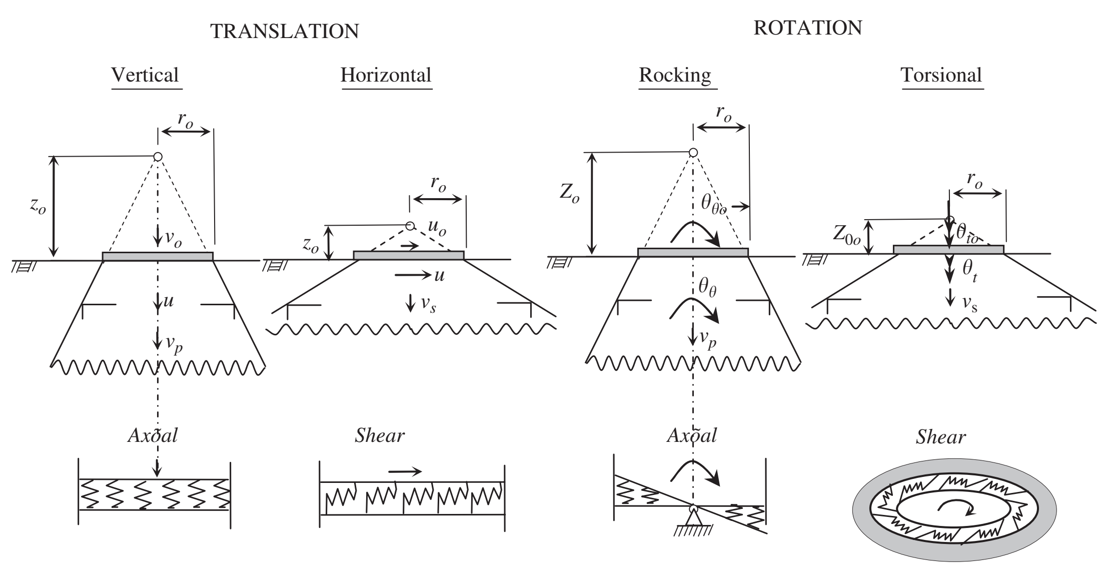

Fig. 3. Cones for various degrees-of-freedom with corresponding aspect ratio (z_0/r_0), wave-propagation velocity and distortion [32,34] vs and vp are the shear and dilatational wave velocities.

displacements with the increasing depth are assumed as in Fig. 3. The shape is also like a truncated cone, for the dynamic loading. Static stiffness of this truncated cone in a circular rigid foundation and equivalent circular foundation can be expressed as in Table 2. At the same time, these stiffness values can be used for rectangular foundations by means of equivalent radii (r_0, r_θ). Finally, the method using these static stiffnesses and accounting for the frequency dependent dynamic stiffness and damping can consider the soil/foundation–structure interaction. It has to be clarified that the equivalent radius works very well for square areas and quite well for rectangular areas with ratios up to 2 [33,35]. For very long foundations, the assumption of an infinite strip foundation may be better suited.

K_v, K_H, K_R and K_T are the vertical, horizontal, rocking and torsional stiffnesses of a foundation, respectively. If it is not preferred to use the equivalent radii for the rectangular and square foundations, one can use formulations given for square and rectangular shapes by Wolf [32,34] or Gazetas [36] or for other arbitrary shape by Dobry and Gazetas [37] and Gazetas and Tassoulas [38].

Since the stiffnesses in the cone model are frequency dependent in the dynamic loading, these static stiffnesses are used for calculating the dynamic stiffness [$S(a_0)$] estimated by the following equation:

$$S(a_0) = K(k(a_0) + ia_0 c(a_0)), \tag{1}$$

where $k(a_0)$ is the dynamic spring coefficient, $c(a_0)$ is the dynamic damping coefficient (including radiation damping) and a_0 is the dimensionless frequency, which is equal to $\omega r_0/v_s$ where ω is the excitation frequency and v_s is the shear velocity of the soil medium.

The dynamic coefficients [$k(a_0)$, $c(a_0)$] of the translational cone and [$k_\theta(a_0)$, $c_\theta(a_0)$] of the rotational cone for a rigid foundation resting on the surface of half-space could be estimated as

$$k(a_0) = 1 - \frac{\mu}{\pi}\frac{z_0}{r_0}\frac{v_s^2}{v^2}a_0^2 \quad c(a_0) = \frac{z_0}{r_0}\frac{v_s}{v}, \tag{2}$$

$$k_\theta(a_0) = 1 - \frac{4}{3}\frac{\mu_\theta}{\pi}\frac{z_0}{r_0}\frac{v_s^2}{v^2}a_0^2 - \frac{1}{3}\frac{a_0^2}{(r_0 v/z_0 v_s)^2 + a_0^2}.$$

$$c_\theta(a_0) = \frac{z_0 v_s}{3 r_0 v}\frac{a_0^2}{(r_0 v/z_0 v_s)^2 + a_0^2} \tag{3}$$

To determine the dimensions and the dynamic characteristic of the cone, an equivalent radius is calculated first. Then aspect ratio of the cone and the wave velocity due to Poisson's ratio should be determined by the equations given in Table 2. It should be noted that for $1/3 < \upsilon \leqslant 1/2$, soil is nearly incompressible. This behavior corresponds to trapped soil beneath the foundation, which moves as a rigid body in phase with the foundation. A close match is achieved by defining the trapped mass (ΔM) to be determined from Table 3.

Here, it must be stated that in the intermediate and higher frequency ranges, the dynamic stiffness coefficient is governed by the damping coefficient, as $c(a_0)$ is multiplied by a_0 in contrast to $k(a_0)$. Both $c_v(a_0)$ and $c_H(a_0)$ of the cone model produce very accurate results in this frequency range [32,34]. Whereas in the lower-frequency range ($a_0 < 2$) and for $\upsilon \leqslant 1/3$, which is of practical importance, the cone's results overestimate (radiation) damping, especially in the vertical translation mode [32,34] (Table 4).

Table 2
Static stiffness values of rigid equivalent-circular foundation, G: shear modulus, r_0: radius of a circular foundation, υ: Poisson ratio, e: embedment height

Stiffness	r_0	No-embedment	Embedment (Emb)
Vertical (K_v)	$\sqrt{\frac{ab}{\pi}}$	$\frac{4Gr_0}{1-\upsilon}$	$\frac{4Gr_0}{1-\upsilon}\left(1+0.54\frac{e}{r_0}\right)\left(1+\left(0.85-0.28\frac{e}{r_0}\right)\right)$
Horizontal (K_H)	$\sqrt{\frac{ab}{\pi}}$	$\frac{8Gr_0}{2-\upsilon}$	$\frac{8Gr_0}{2-\upsilon}\left(1+\frac{e}{r_0}\right)$
Rocking (K_R)	$\sqrt[4]{\frac{a^3 b}{3\pi}}$	$\frac{8Gr_0^3}{3(1-\upsilon)}$	$\frac{8Gr_0^3}{3(1-\upsilon)}\left[1+2.3\frac{e}{r_0}+0.58\left(\frac{e}{r_0}\right)^3\right]$
Torsional (K_T)	$\sqrt[4]{\frac{ab(a^2+b^2)}{6\pi}}$	$\frac{16Gr_0^3}{3}$	$\frac{16Gr_0^3}{3}\left(1+2.67\frac{e}{r_0}\right)$

Table 3
Properties of cone models [32]

Motion	Aspect ratio (z_0/r_0)	Poisson's ratio (υ)	Wave velocity (v)	Trapped mass (ΔM)
Horizontal	$\frac{\pi}{8}(2-\upsilon)$	all (υ)	v_s	0
Vertical	$\frac{\pi}{4}(1-\upsilon)\left(\frac{v}{v_s}\right)^2$	$\upsilon \leqslant 1/3$	v_p	0
		$1/3 < \upsilon \leqslant 1/2$	$2v_s$	$2,4(\upsilon - 1/3)\rho A_0 r_0$
Torsional	$\frac{9\pi}{32}$	all (υ)	v_s	0
Rocking	$\frac{9\pi}{32}(1-\upsilon)\left(\frac{v}{v_s}\right)^2$	$\upsilon \leqslant 1/3$	v_p	0
		$1/3 < \upsilon \leqslant 1/2$	$2v_s$	$1,2(\upsilon - 1/3)\rho I_0 r_0$

2.3. Proposed model and analysis procedure to fluid–rectangular tank–soil/foundation system

Seismic behavior of fluid–rectangular tank–soil/foundation system is a complex problem. The problem can simply be idealized as in Fig. 4. In this idealization, fluid is modeled with Housner's two mass representations and the soil/foundation system is modeled with cone model. For fluid, only one convective mode is taken into account, and for soil/foundation system, only horizontal and rocking motion is considered in the model.

As aimed in this paper, to consider the fluid–structure–soil foundation interaction and to determine overall the system response to the dynamic excitation, initially, the system must be represented by four properties (Fig. 5). First, one is lateral stiffness of the supporting structure (k). This stiffness of the typical rectangular tanks can be determined with well-known equations of the shell or the first mode effective stiffness of the system (k_1^*) simply calculated as suggested in EC 8 [26]. This code proposed the period of vibration of the first impulsive storage tank–fluid horizontal mode is given approximately by

$$T_{\rm f} = 2\pi\sqrt{d_{\rm f}/g}, \tag{4}$$

where $d_{\rm f}$ is the deflection of the wall on the vertical center line and at the height of the impulsive mass, when the wall uniformly loaded in the direction of the ground motion and of magnitude $m_{\rm i}g/(4BH)$. Where B is the half-width perpendicular to the direction of loading (earthquake direction) and $m_{\rm i}$ is the impulsive mass (see Table 1). The second property is the stiffness $k_{\rm cn}$ for the convective mass given by Housner (see Table 1). The third one is the mass m_1, which is equal to convective mass $m_{\rm c}$ estimated from Table 1 as well. Finally, the fourth one is the mass m_2, which is summation of the impulse mass, the total mass of wall of the rectangular tank perpendicular to

Table 4
Properties of the considered soil types

Soil types	ζ_g	E (kN/m^2)	G (kN/m^2)	E_c (kN/m^3)	γ (kg/m)	υ	v_s (m/s)	v_p (m/s)
S1	5.00	7,000,000	269,2310	9,423,077	2000	0.30	1149.1	2149.89
S2	5.00	2,000,000	769,230	2,692,308	2000	0.30	614.25	1149.16
S3	5.00	500,000	192,310	673,077	1900	0.35	309.22	643.68
S4	5.00	150,000	57,690	201,923	1900	0.35	169.36	352.56
S5	5.00	75,000	26,790	160,714	1800	0.40	120.82	295.95
S6	5.00	35,000	12,500	75,000	1800	0.40	82.54	202.18

E: Young modulus, G: Shear Modulus, E_c: Bulk modulus, υ: Poisson's ratio, γ: unit density of the soil.

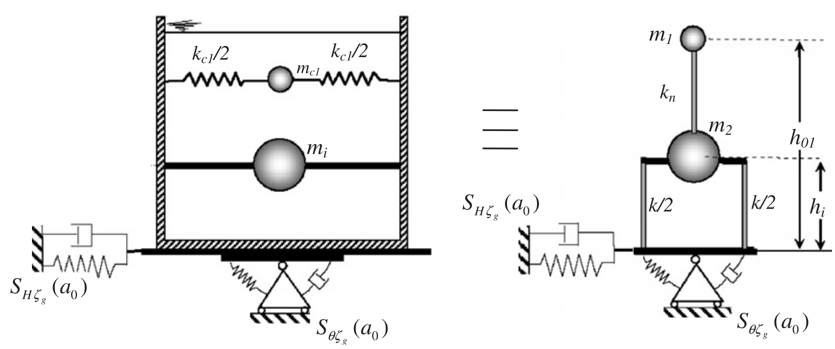

Fig. 4. Considered mechanical model for the fluid–rectangular tank–soil/foundation system.

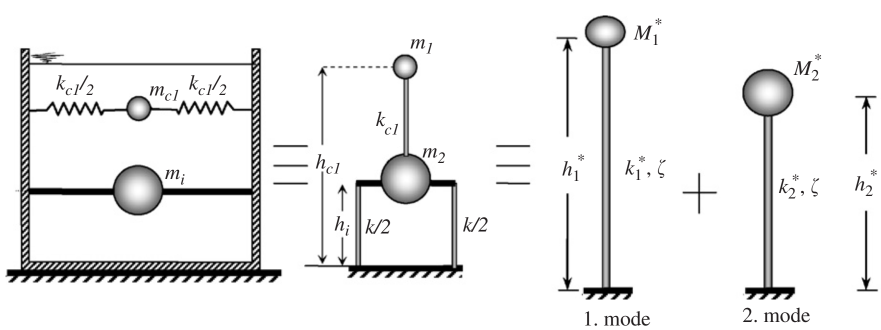

Fig. 5. Two mass model represented by two mode properties of the elevated tank.

the excitation. Modal properties such as effective modal mass, heights, and stiffness can be calculated from this two degree-of-freedom system (Fig. 5).

Where M_1^* & M_2^*; h_1^* & h_2^*; k_1^* & k_2^* are the efffecive masses, effective heights and effective stiffnesses of the first and the second modes, respectively. These modal properties can be estimated using Eqs. (5) and (6) [39].

$$M_n^* = \Gamma_n L_n^h = \frac{(L_n^h)^2}{M_n}; \quad h_n^* = \frac{L_n^\theta}{L_n^h}; \quad k_n^* = \omega_n^2 M_n^*, \tag{5}$$

where

$$M_n = \phi_n^t M \phi_n = \sum_{i=1}^{N} m_i \phi_{jn}^2 : \quad \Gamma_n = \frac{L_n^h}{M_n}:$$
$$L_n^h = \sum_{i=1}^{N} m_i \phi_{jn} : \quad L_n^\theta = \sum_{i=1}^{N} h_i m_i \phi_{jn}. \tag{6}$$

where N is the total mode number, which is considered, ϕ_n is the mode vector of the nth mode and ω_n^2 is the eigenvalue of the nth mode. Thus, the model can be represented with two single-degree-of-freedom systems. Because of the absolute differences between the sloshing stiffness k_{cn} and the stiffness of the supporting system k, it should be assumed that the first mode represents the sloshing and the second one the impulsive mode, respecively.

In case of soil Poisson ratio for soil ($\upsilon \leqslant 1/3$) and ($1/3 < \upsilon \leqslant 1/2$), the dynamic properties of single-degree-of-freedom system mentioned above has to be calculated separately. Because, the trapped or additional mass (ΔM) must be considered in case of $1/3 < \upsilon \leqslant 1/2$ (Fig. 6). Except for some special circumstances, the relatively more important motions such as the horizontal and the rocking, would be adequate for analyzing the fluid–rectangular tank–foundation/soil systems. Finally, calculated internal forces or displacement determined can be combined with any one of the modal combination techniques i.e. ABS, SRSS, CQC, etc.

For the system given in Fig. 6, the force–displacement relationship in the horizontal direction and the moment-rotation relationship in the rocking interaction can be formulated in frequency domain as

$$\begin{aligned} P_0(\omega) &= K_{\mathrm{H}} k_{\mathrm{H}\zeta_{\mathrm{g}}}(a_0) u_0(\omega) + \frac{r_0}{v_{\mathrm{s}}} K_{\mathrm{H}} c_{\mathrm{H}\zeta_{\mathrm{g}}}(a_0) \dot{u}_{0_{\mathrm{o}}}(\omega) \\ &= S_{\mathrm{H}\zeta_{\mathrm{g}}}(a_0) u_{\mathrm{b}}(\omega), \end{aligned} \tag{7}$$

$$\begin{aligned} M_0(\omega) &= K_\theta k_{\theta\zeta_{\mathrm{g}}}(a_0) \theta_{\mathrm{b}}(\omega) + \frac{r_0}{v_{\mathrm{s}}} K_\theta c_{\theta\zeta_{\mathrm{g}}}(a_0) \dot{\theta}_{\mathrm{b}}(\omega) \\ &= S_{\theta\zeta_{\mathrm{g}}}(a_0) \theta_{\mathrm{b}}(\omega), \end{aligned} \tag{8}$$

where $k_{\mathrm{H}\zeta_{\mathrm{g}}}$, $c_{\mathrm{H}\zeta_{\mathrm{g}}}$, $k_{\theta\zeta_{\mathrm{g}}}$ and $c_{\theta\zeta_{\mathrm{g}}}$ are the dynamic stiffness coefficients including ground damping effects of the foundation/soil system. The dynamic stiffnesses $[(S_{\mathrm{H}\zeta_{\mathrm{g}}}(a_0))$ and $(S_{\theta\zeta_{\mathrm{g}}}(a_0))]$ for the horizontal and rocking motions depending on the dimensionless frequency are approximated as below

$$S_{\mathrm{H}\zeta_{\mathrm{g}}}(a_0) = K_{\mathrm{H}} k_{\mathrm{H}\zeta_{\mathrm{g}}}(a_0)(1 + 2\mathrm{i}\zeta_{\mathrm{H}}(a_0) + 2\mathrm{i}\zeta_{\mathrm{g}}) \tag{9}$$

$$S_{\theta\zeta_{\mathrm{g}}}(a_0) = K_\theta k_{\theta\zeta_{\mathrm{g}}}(a_0)(1 + 2\mathrm{i}\zeta_\theta(a_0) + 2\mathrm{i}\zeta_{\mathrm{g}}), \tag{10}$$

where ζ_{g} is the material damping ratio of the soil, $\zeta_{\mathrm{H}}(a_0)$ and $\zeta_\theta(a_0)$ are the radiation-damping ratios for the horizontal and rocking motion of the soil to be estimated as

$$\zeta_{\mathrm{H}}(a_0) = \frac{a_0 c_{\mathrm{H}}(a_0)}{2k_{\mathrm{H}}(a_0)}, \quad \zeta_\theta(a_0) = \frac{a_0 c_\theta(a_0)}{2k_\theta(a_0)}. \tag{11}$$

For a system of three-degrees-of-freedom, the dynamic equilibrium can be formulated for $\upsilon \leqslant 1/3$ as

$$\begin{bmatrix} \frac{\omega_{sn}^2}{\omega^2}(1+2\mathrm{i}\zeta) - 1 & -1 & -1 \\ -1 & \frac{S_{\mathrm{H}\zeta_{\mathrm{g}}}(a_0)}{M_n^* \omega^2} - 1 & -1 \\ -1 & -1 & \frac{S_{\vartheta\zeta_{\mathrm{g}}}(a_0)}{M_n^* h^2 \omega^2} - 1 \end{bmatrix} \begin{Bmatrix} u(\omega) \\ u_{\mathrm{b}}(\omega) \\ h\theta_{\mathrm{b}}(\omega) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} u_{\mathrm{g}}(\omega). \tag{12}$$

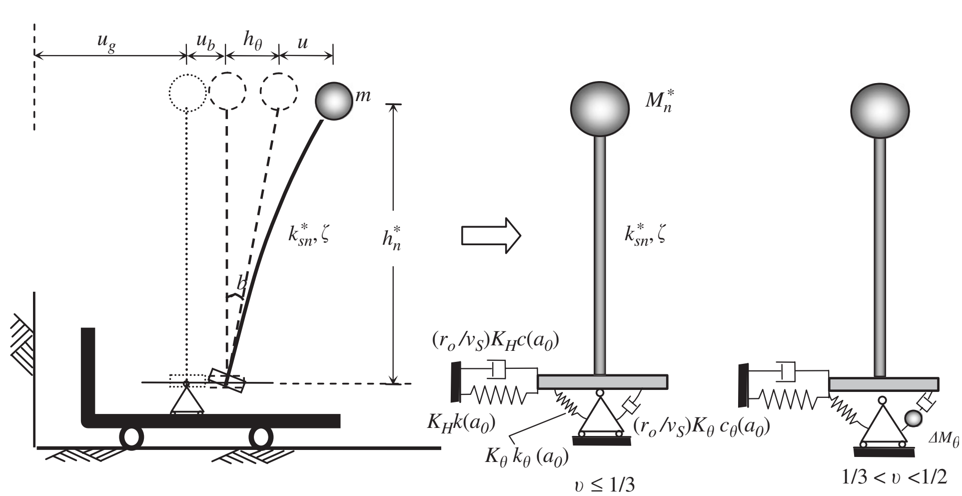

Fig. 6. Dynamic model of structure and soil for horizontal and rocking motions.

If $u_b(\omega)$ and $h\theta_b(\omega)$ are expressed using $u(\omega)$, Eq. 12 yield the following equation

$$u(\omega)=\frac{1}{\left[\frac{1}{\omega^2}-\frac{M_n^*}{S_{\mathrm{H}\zeta_g}(a_0)}-\frac{\omega^2 M_n^* h_n^{*2}}{S_{\theta\zeta_g}(a_0)}-\frac{1}{\omega_{sn}^2(1+2i\zeta)}\right](1+2i\zeta)\omega_{sn}^2}u_g(\omega). \tag{13}$$

If the same equations were written for $1/3<\upsilon\leqslant 1/2$ Eq. (12) could be written as

$$\begin{bmatrix} \frac{\omega_{sn}^2}{\omega^2}(1+2i\zeta)-1 & -1 & -1 \\ -1 & \frac{S_{\mathrm{H}\zeta_g}(a_0)}{M_n^*\omega^2}-1 & -1 \\ -1 & -1 & \frac{S_{\vartheta\zeta_g}(a_0)}{M_n^* h^2\omega^2}-\frac{\Delta M_\theta}{M_n^* h^2}-1 \end{bmatrix}\begin{Bmatrix} u(\omega) \\ u_b(\omega) \\ h\theta_b(\omega)\end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \\ 1\end{Bmatrix}u_g(\omega), \tag{14}$$

where $u_b(\omega)$ and $h\theta_b(\omega)$ are expressed using $u(\omega)$ as

The lateral displacement depending on the structural rigidity was derived using Eq. (14) in the incompressible soil as below:

$$u(\omega)=\frac{u_g(\omega)}{\omega_{sn}^2(1+2i\zeta)\left[\frac{1}{\omega^2}-\frac{1}{\omega_{sn}^2(1+2i\zeta)}-\frac{M_n^*}{S_{\mathrm{H}\zeta_g}(a_0)}-\frac{1}{(S_{\vartheta\zeta_g}(a_0)/M_n^* h^2)-(\Delta M_\theta/M_n^* h^2)\omega^2}\right]}, \tag{15}$$

where ω_{sn}^2 $(=k_{sn}^*/M_n^*)$ is the square of the angular frequency of the fixed base single-degree-of-freedom system, $u_g(\omega)$ is the effective input motion in the frequency domain. It may be obtained by means one of the transformation techniques like Fourier and/or Laplace i.e. using Fourier transformation, the displacements in the time domain $u_g(t)$ can be transformed in the frequency domain. In addition, the solution in the frequency domain can be transformed in the time domain using inverse Fourier transformation.

3. Numerical examples

In this study, a reinforced concrete rectangular storage tanks with two different wall thicknesses are considered as shown in Fig. 7. The first one has the 0.5 m wall thickness named as flexible tank and second has 1 m wall thickness named as rigid tank. These tanks are selected as the same tank investigated by Koh et al. [17] and Doğangün and Livaoğlu [15]. In the examples, Young's modulus, the weight of concrete per unit volume and density of fluid are taken to be 32,000 MPa and 25 kN/m^3 and 1000 kg/m^3, respectively. The other characteristics like dimensions of the tank and the foundation system are shown in Fig. 7.

In the seismic analyses, it is assumed that tanks are subjected to two different earthquakes separately. North-South component of the August 17, 1999 Kocaeli Earthquake Yarimca record and East–West component of the November 12, 1999 Düzce Earthquake Meteorology Building record are used (see Fig. 8). First 20 s of both ground accelerations were taken into consideration in the analyses. To evaluate variations of the dynamic parameters in the tanks depending on different soil conditions, six soil types as were used (see Table 3). Soil conditions recommended in the literature are taken into account in the selection of the soil types and their properties [40,41]. For two different wall thicknesses and six different soil types, seismic analyses of the tank and soil systems were performed in cases of no embedment ($e/r_0=0$) and embedment ($e/r_0=1$).

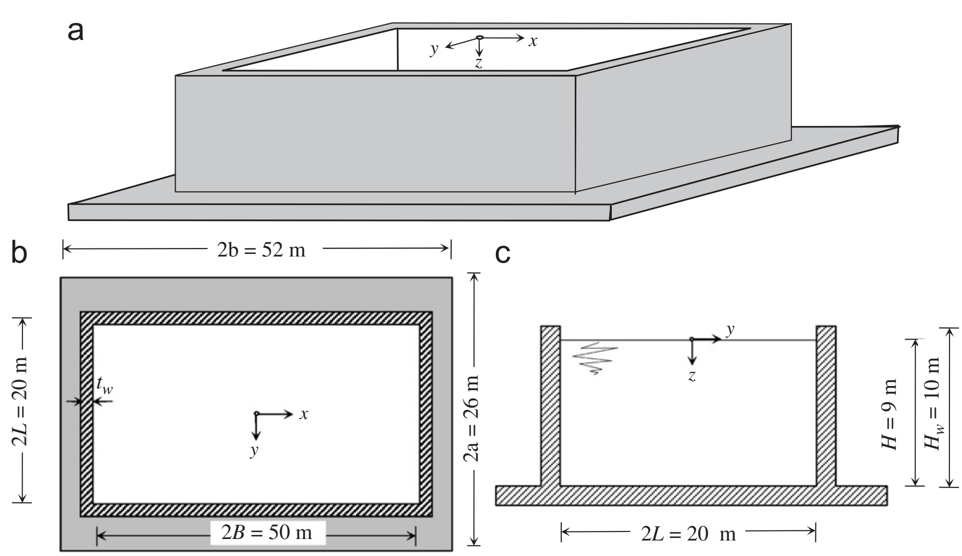

Fig. 7. Plan and elevation of the sample rectangular tank.

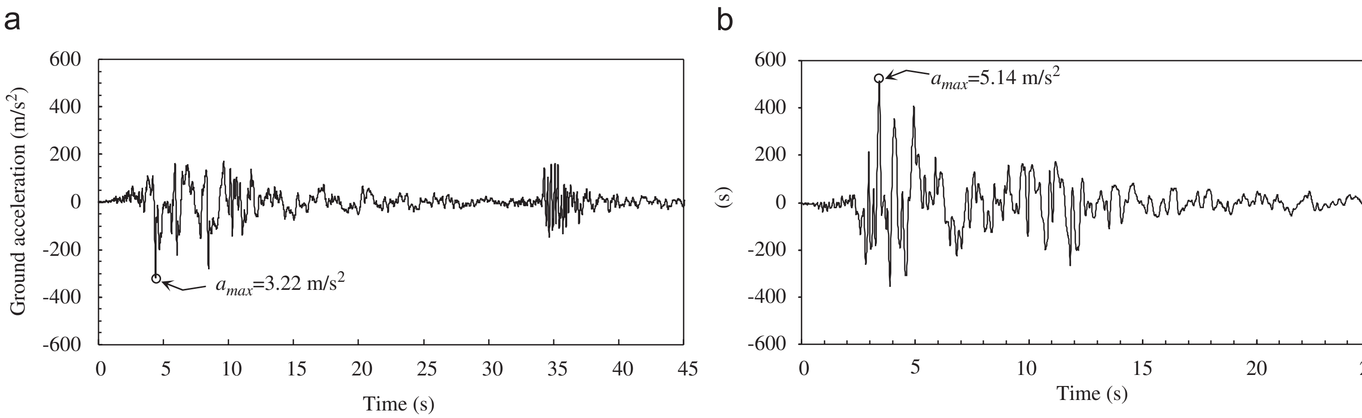

Fig. 8. Considered ground motions from (a) November Duzce, N-S component of Yarimca station and (b) August Kocaeli Earthquakes, E-W component of Düzce meterology station in Turkey.

Table 5
Obtained maximum results of displacement of the tank at the height of the impulsive mass and their time

	S1		S2		S3		S4		S5		S6	
	t(s)	*u*(m)	*t*(s)	*u*(m)	*t*(s)	*u*(m)	*t*(s)	*u*(m)	*t*(s)	*u*(m)	*t*(s)	*u*(m)
Flexible tanks (*for August, 19 1999 Kocaeli Earthquake*)												
$e/r_0 = 0$	9.98	−0.099	9.99	−0.100	10.01	−0.107	12.93	0.097	13.53	0.083	13.00	0.066
$e/r_0 = 1$	9.98	−0.099	9.98	−0.100	9.99	−0.102	10.03	−0.102	12.92	0.096	12.96	0.080
Flexible tanks (*for November, 12 1999 Düzce Earthquake*)												
$e/r_0 = 0$	18.27	0.097	18.27	0.097	18.24	0.103	14.48	0.133	14.58	−0.125	14.60	−0.117
$e/r_0 = 1$	18.27	0.097	18.27	0.097	18.25	0.098	18.25	0.109	14.48	−0.130	14.56	−0.120
Rigid tanks (*for August, 19 1999 Kocaeli Earthquake*)												
$e/r_0 = 0$	11.50	0.022	9.89	−0.019	6.73	0.012	6.73	0.014	6.73	0.014	6.73	0.015
$e/r_0 = 1$	11.49	0.022	10.79	−0.021	9.89	−0.014	6.73	0.013	6.73	0.014	8.73	0.014
Rigid tanks (*for November, 12 1999 Düzce Earthquake*)												
$e/r_0 = 0$	6.41	−0.022	6.41	−0.024	6.43	−0.021	5.62	0.020	5.62	0.018	5.30	−0.015
$e/r_0 = 1$	6.40	−0.022	6.40	−0.022	6.40	−0.023	6.40	−0.019	5.61	0.019	5.61	0.017

4. Discussion of the analysis results

Fourty-eight different analyses with varying combinations of parameters such as soil type, embedment, and for two different ground motions the wall flexibility were analyzed using the suggested procedure. The analyses were carried out via a computer program SAT-2005 [42] coded for the seismic analysis of the ground-level rectangular tanks. The displacement response obtained at the height of the impulsive mass, sloshing response, base shear forces in the wall determined from the analyses performed by means of SAT-2005 are illustrated and discussed later comparatively and also using these results, effects of the wall flexibility and the foundation embedment of the tanks are investigated too.

4.1. Soil–structure interaction effects on the displacement

Using the suggested model, it is possible to determine the displacement response of the system at the height of the impulsive mass. These responses and their variations in time are obtained from the analyzed models, furthermore maximum values of the results from all analyses are given in Table 5 for two different ground motions. The Table 5 shows that maximum displacement responses of the systems are different from each other. Therefore, the peaks were occurred with a time lag of the 10–13 s for the flexible tanks and of the 6.7–11.50 s for the rigid tanks for Kocaeli Earthquake. From the analyses of same configurations using Düzce records, the maximum displacement response are calculated in the time interval of 14–18.5 s for the flexible tank and of 5–6 s for the rigid tank. Tank flexibility effects calculated on the displacement response of the systems reflects a significant SSI influence on the response. Another sign of SSI effect is that for both flexible and rigid tanks, maximum value of the displacement response changed with changing soil conditions. For example in case of no embedment for flexible tanks, value of maximum displacement is 0.099 m for the tank in S1 soil type, where same quantity was calculated as 0.066 m for the tank in S6 soil type due to Kocaeli Earthquake. Furthermore, a similar trend was observed between S1 and S6 soil types due to Düzce Earthquake. Similar comparisons are given in Table 5.

Here it has to be stated that, not all results from analyzed models can be illustrated here. So some comparisons are selected to describe the others for all analysis. From obtained results, the flexible tanks and rigid tank in S1 soil type and S3 soil type are illustrated in Figs. 9 and 10, respectively. Same configurations were shown between flexible tanks on S1 and S6 soil types in Fig. 11. As can be seen from Fig. 9, for flexible tanks one could easily say that the displacement is not affected from SSI for both considered ground motions. Since almost all responses and their occurrence times are almost same. When the comparisons are made for the other system with the soil profiles from S1 to S4, situations are same but the deviations are different from each other. However, for the rigid tank different behavior occurred. If same comparisons are made for the rigid tank as in Fig. 10, it is seen that the deviations are more pronounced. Both maximum response and their deviations in time differ significantly. All the other comparisons have same characteristics and especially in the point of deviation rate, SSI

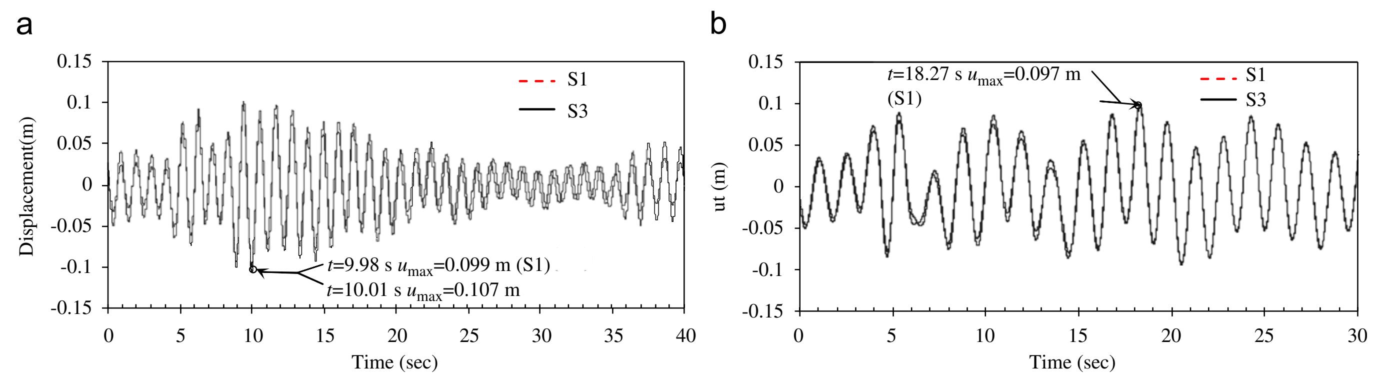

Fig. 9. Deviations of the displacements in time between the flexible rectangular tank on S1 soil type and S3 soil type for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

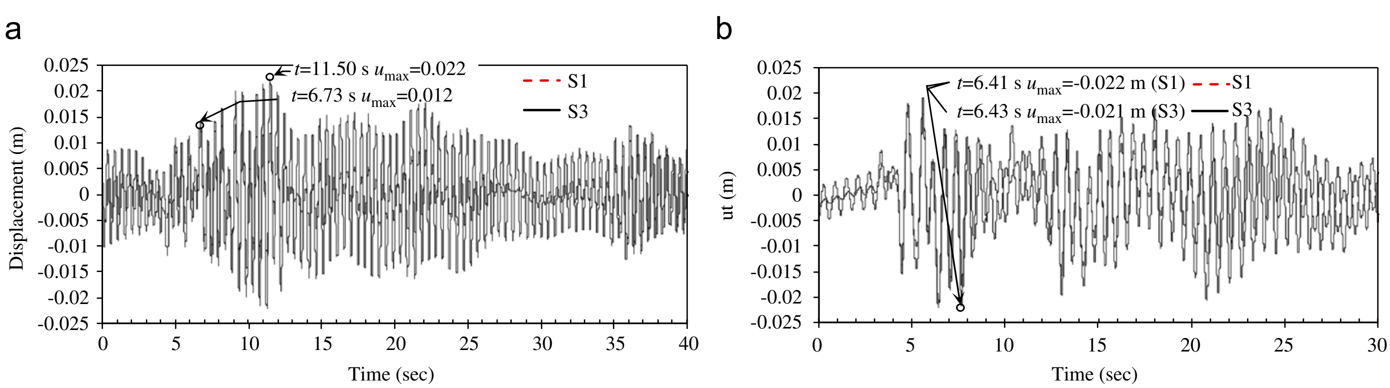

Fig. 10. Deviations of the displacements in time between the rigid-rectangular tank on S1 soil type and S3 soil type for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

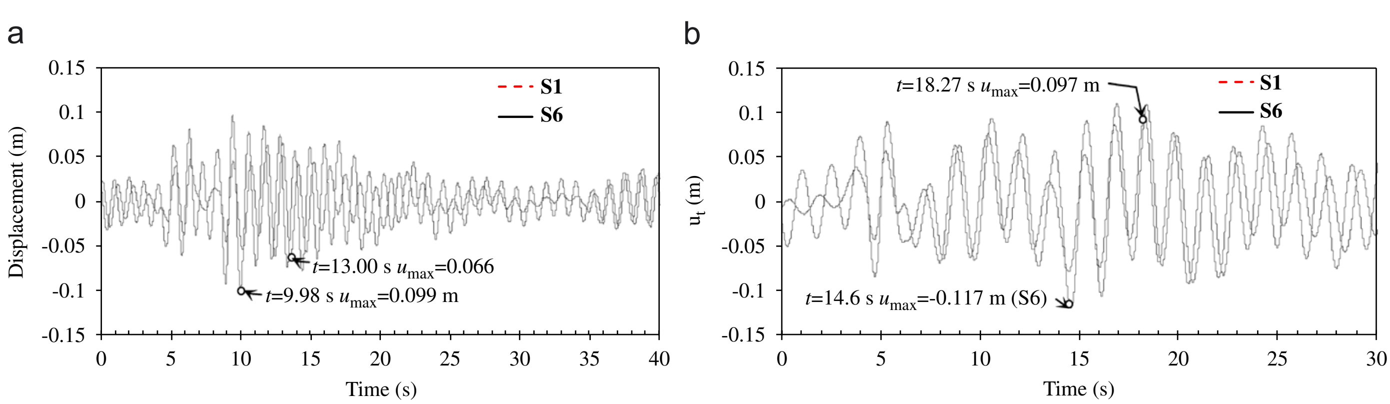

Fig. 11. Deviations of the displacements in time between the flexible rectangular tank on S1 soil type and S6 soil type for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

is more effective on rigid-rectangular tanks for all soil type investigated in this paper, but the magnitudes of displacements are so small that these changes can be ignored for these rigid wall systems from a practical design point of view.

Instead, displacement response of the flexible tanks is only affected in case of comparisons observed between S1 and S5 or S6 soil types, i.e. time deviations of the displacements in between S1 and S6 soil types are illustrated in Fig. 11. One can be seen in Fig. 11 that the SSI affected the system behavior so that, displacement response decreased almost 33%. Time deviations of displacements for the rigid tanks are different and are not remarkably changed, but displacement amplitudes are decreased due to SSI. Furthermore, the decreases as much as 0.033 m for ground motion of Kocaeli 1999 Earthqukae recorded in Yarımca station can cause the tank failure during an earthquake. Differently for the Meteorolgy building record of the Düzce Earthquake, time history response of the displacement show that soft soil condition increased the displacement response approximately as much as 20%. For the other conditions, however, similar responses as for Kocaeli Earthquake are obtained.

4.2. Soil–structure interaction effects on the sloshing displacement

The maximum sloshing displacements estimated from 48 different analyses are given in Table 6. As Veletsos and Tang [43] pointed out for cylindrical tanks, from the results obtained in this study showed that neither maximum sloshing displacement nor their occurrence times are not considerably changed for the investigated rectangular tanks. In addition, from the results it is concluded that the maximum sloshing occurs as 0.5 m in 7.17 s for almost all systems excited by Yarımca record of Kocaeli 1999 Earthquake and as interval of 0.75–0.78 m in 17.76 s for Meteorology building record of Düzce 1999 Earthquake. It is appropriate to say that the maximum sloshing displacements of the tanks due to both considered ground motions gave significantly different responses. As can be seen from the illustrations and the maximum response values submitted in Table 6, for Düzce records these values can be as high as 0.785 m, while they were calculated as 0.505 m for Kocaeli Earthquake. However, author concluded from the results that it is not worth to consider the wall flexibility effects on sloshing displacements.

For the amplitude of the sloshing, the deviations between S1 to S6 showed that response is not different from each other. These devastations are illustrated for flexible tanks in Fig. 12 and for rigid tanks in Fig. 13. It was seen from these two figures that SSI does not have any considerable effects on sloshing responses both for rigid or flexible tanks. Especially when the sloshing responses compared with displacements response, deviations are very small, i.e. it is clearly shown from the result of all analyses and for both ground motions that maximum changes reaches 4–5% for the rigid tank between S1 and S6 soil types.

4.3. Soil–structure interaction effects on the base shear

For all analyses, the estimated maximum base shear forces and their occurrence times in case of embedment, soil conditions and two ground motions for two different types of rectangular tanks are given in Table 7. Maximum base shear responses for Kocaeli Earthquake were calculated at 4.89–8.96 s and for Düzce Earthquake at 4.28–4.33 s as can be seen from these results. The results also stated that this response decreases with decreasing stiffness of the soil foundation system. This tendency can be seen for both flexible and rigid tanks.

The deviations of the base shear forces in time are illustrated and compared between S1 and S6 soil types for the flexible tanks in Fig. 14 and rigid tanks in Fig. 15. Both illustrations have different characteristic and describe different behavior of the tank. For example, for almost all flexible tanks the deviation of the base shear is coincide

Table 6
Obtained maximum results of sloshing displacement and their occurring times

Soil Type	S1		S2		S3		S4		S5		S6	
	t(s)	u_s(m)	t(s)	u_s(m)	t(s)	u_s(m)	t(s)	u_s(m)	t(s)	u_s(m)	t(s)	u_s(m)
Flexible Tanks (for August, 19 1999 Kocaeli Earthquake)												
$e/r_0=0$	7.15	0.488	7.15	0.487	7.16	0.486	7.16	0.485	7.17	0.475	7.17	0.465
$e/r_0=1$	7.16	0.488	7.16	0.487	7.16	0.487	7.16	0.486	7.16	0.484	7.17	0.479
Flexible Tanks (for November, 12 1999 Düzce Earthquake)												
$e/r_0=0$	17.76	0.785	17.76	0.79	17.76	0.783	17.76	0.78	17.66	0.766	17.71	0.750
$e/r_0=1$	17.76	0.785	17.76	0.79	17.76	0.784	17.76	0.782	17.63	0.777	17.66	0.769
Rigid Tanks (for August, 19 1999 Kocaeli Earthquake)												
$e/r_0=0$	7.13	0.505	7.13	0.504	7.13	0.503	7.14	0.499	7.14	0.490	7.15	0.480
$e/r_0=1$	7.13	0.505	7.13	0.504	7.13	0.504	7.14	0.503	7.14	0.499	7.14	0.494
Rigid Tanks (for November, 12 1999 Düzce Earthquake)												
$e/r_0=0$	17.6	0.752	17.6	0.752	17.6	0.751	17.6	0.75	17.66	0.747	17.71	0.741
$e/r_0=1$	17.6	0.752	17.6	0.752	17.6	0.752	17.6	0.751	17.63	0.75	17.66	0.747

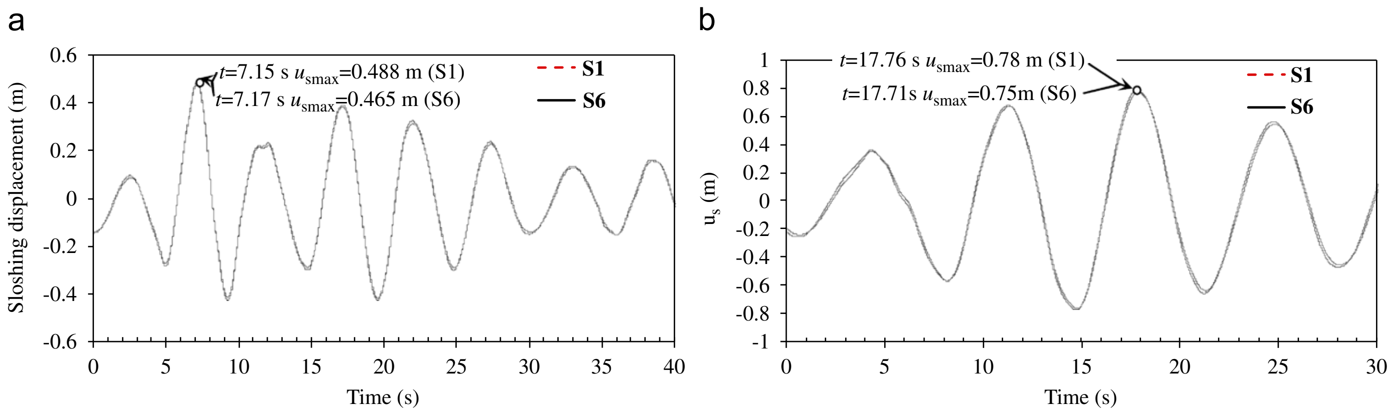

Fig. 12. Deviations of the sloshing displacements in time between the flexible tank on S1 and S6 soil types for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

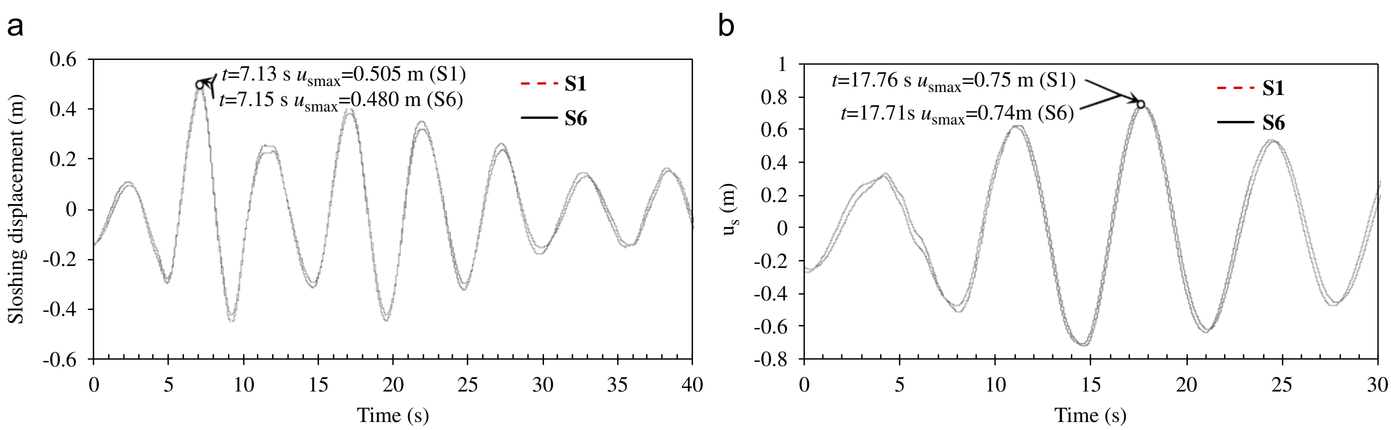

Fig. 13. Deviations of the sloshing displacements in time between the rigid tank on S1 and on S6 soil types for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

Table 7
Obtained maximum results of base shear forces and their occurring times

	S1		S2		S3		S4		S5		S6	
	t(s)	V_b(kN)	t(s)	V_b(kN)	t(s)	V_b(kN)	t(s)	V_b(kN)	t(s)	V_b(kN)	t(s)	V_b(kN)
Flexible Tanks (for August, 19 1999 Kocaeli Earthquake)												
$e/r_0=0$	8.96	−45,775	8.96	−45,874	8.96	−47,271	8.96	−41,476	8.96	−38,520	8.96	−37,289
$e/r_0=1$	8.96	−45,763	8.96	−45,798	8.96	−46,163	8.96	−46,605	8.96	−42,424	8.96	−39,353
Flexible Tanks (for November, 12 1999 Düzce Earthquake)												
$e/r_0=0$	4.33	55,264	4.33	55,689	4.33	57,849	4.33	56,326	4.33	52,108	4.33	52,832
$e/r_0=1$	4.33	55,174	4.33	55,404	4.33	56,417	4.33	57,956	4.33	55,882	4.33	52,996
Rigid Tanks (for August, 19 1999 Kocaeli Earthquake)												
$e/r_0=0$	8.95	−43,333	8.96	−44,248	4.88	−40,400	4.89	−37,755	4.88	−36,837	4.88	−35,021
$e/r_0=1$	4.89	−44,539	8.95	−44,176	8.96	−40,113	4.88	−39,565	4.88	−38,803	4.88	−37,406
Rigid Tanks (for November, 12 1999 Düzce Earthquake)												
$e/r_0=0$	4.33	47,122	4.28	48,448	4.28	49,067	4.28	49,055	4.28	48,585	4.28	47,951
$e/r_0=1$	4.33	48,033	4.28	46,991	4.28	49,470	4.28	49,107	4.28	49,153	4.28	48,670

but amplitude of the base shear response differs from each other. While, the stiffness of the soil/foundation system decreases, the base shear amplitude decreases. Contrary to the sloshing displacements, SSI can play an important role on the response of base shear as it decreases the response 19% (see Fig. 14). Same expressions can be repeated for the rigid tanks investigated in this paper, but the deviations in time are different except

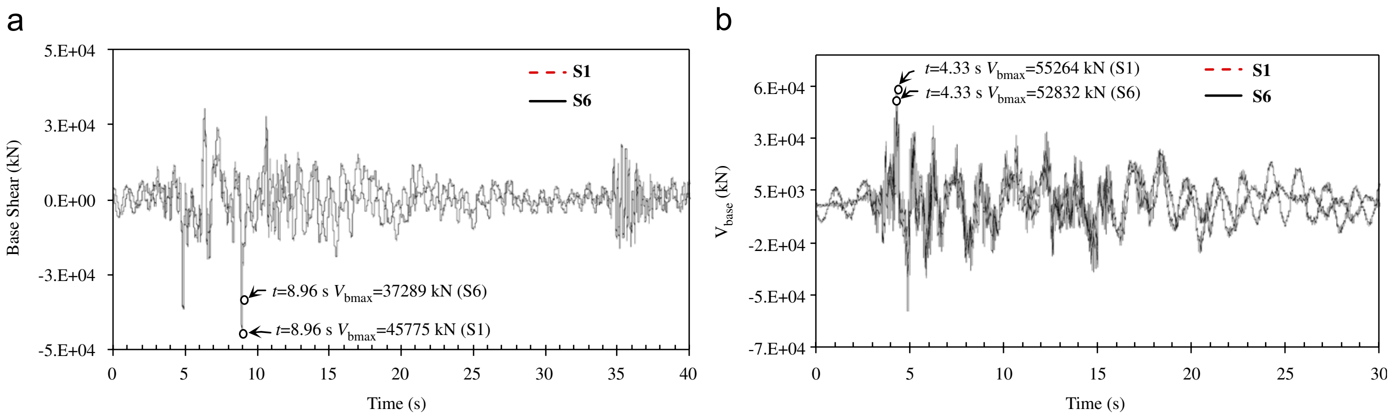

Fig. 14. Deviations of the base shear forces in time between the flexible rectangular tank on S1 and S6 soil types for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

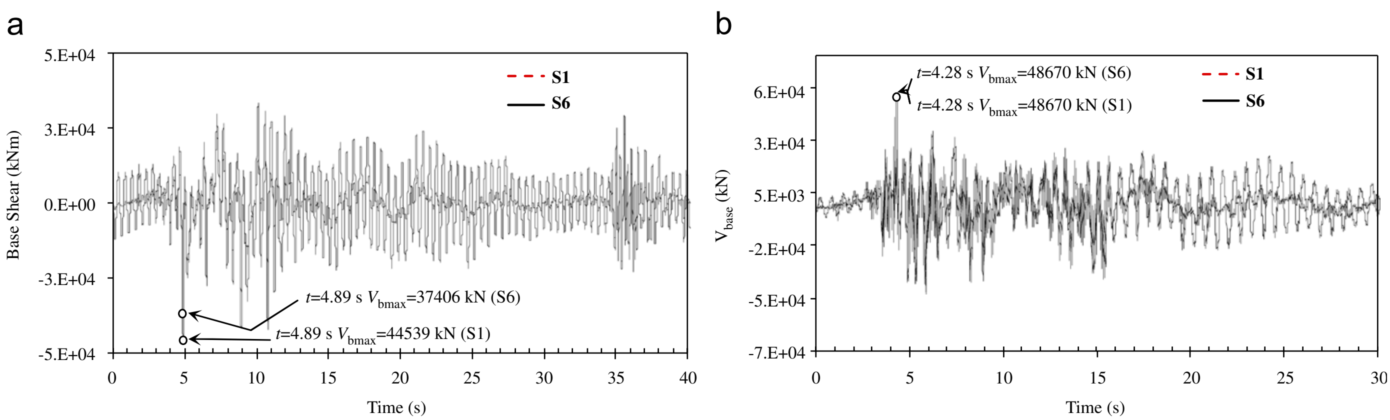

Fig. 15. Deviations of the base shear forces in time between the rigid-rectangular tank on S1 and S6 soil types for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

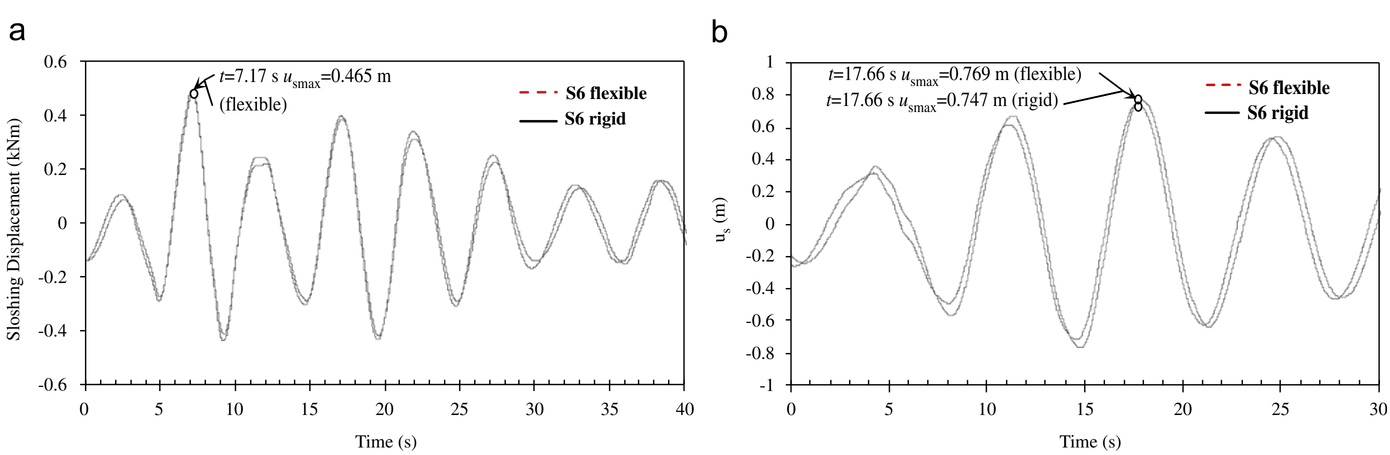

Fig. 16. Deviations of the sloshing displacements in time between the flexible rectangular tank and rigid tank on S6 soil type for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

for the cases between S1 to S2. When the other comparisons are made, it can be seen that the same deviations given in Fig. 15 that for the tank in S6 soil type has different behavior than the other tanks in soil types such as like S1 and S2.

4.4. *Effects of the wall flexibility*

From the all comparisons made before, effects of the wall flexibility can be easily observed, some of which are given below in Fig. 16 for sloshing and in Figs. 17 and 18 for the base shear forces. When the comparisons are made for the displacement at the height of impulsive mass, the fact that the decreases occur in response can be clearly indicated from the results and easily predicted, without doing any analysis. These decreases reached as high as 90% in this study.

Similarly, from the comparisons one can conclude that the wall flexibility does not remarkably affect the sloshing response of the systems (see Fig. 16). For almost all systems, same deviations were encountered in time and caused a similar decreasing trend with the magnitude of almost 3%. This decrease is not significant for the sloshing displacement behavior of the rectangular tanks.

When the similar comparisons are made for the base shear, the maximum values of the response and seismic behavior of the system differ from the each other. Although the considered mass of the walls for rigid tanks is two times larger than that of flexible ones, flexible tanks have almost same maximum values of the base shear forces for the stiff soil type as seen in Fig. 17. Smaller values for the stiff soil types (i.e. for the S6 soil type Fig. 18) it is shown that the decreases to the value of the base shear for the systems excited by Yarımca record of Kocaeli 1999 Earthquakes, were calculated as 7% and similarly for Düzce Earthquake this deviation reached up to 8%. This decrease reached up to 17% for the S3 soil type in case of no embedment and for the S4 soil type in case of embedment.

4.5. Effects of the foundation embedment

All model considered in this study are analyzed in case of embedment and no embedment. The results from these analyses (see Tables 5–7) show that especially for the sloshing response, embedment of the system do not play an important role on the behavior. However, for the displacement and base shear response situation it is not the case. The displacement response of the tank wall is particularly affected for the case of rigid tank system. For example, the deviations in the rigid tank induced by Kocaeli Earthquake in S3 reaches 14%, but the rate may cause to misleading interpretation of results so that the decrease in magnitude is only 0.002 m. Furthermore, the decrease rate can reach up to 17% for the flexible tank in S6 soil type. Additionally, the decreases due to embedment show increasing trend, when soil stiffness decreases.

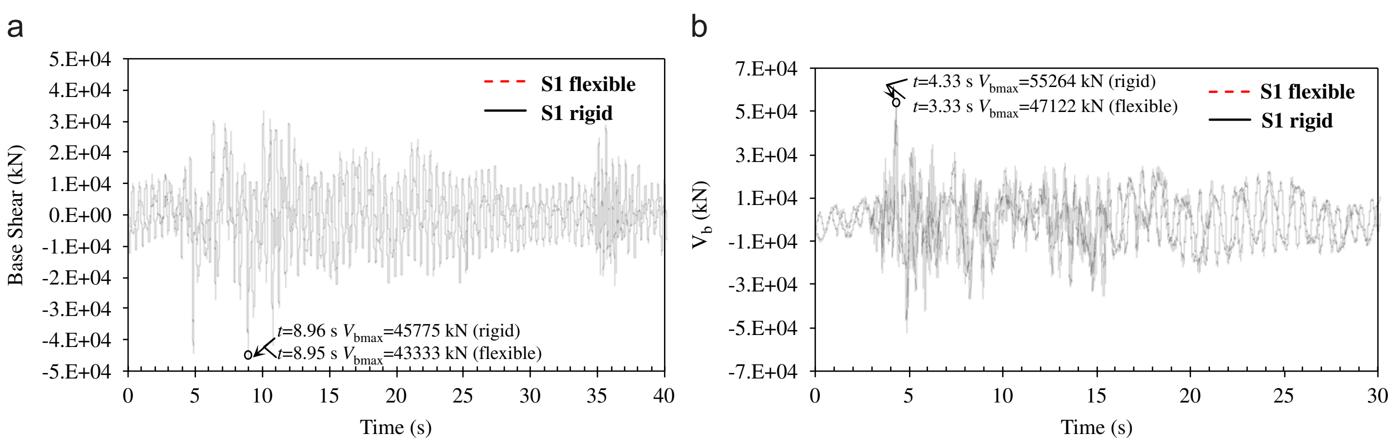

Fig. 17. Deviations of the base shear forces in time between the flexible rectangular tank and rigid tank on S1 soil type for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

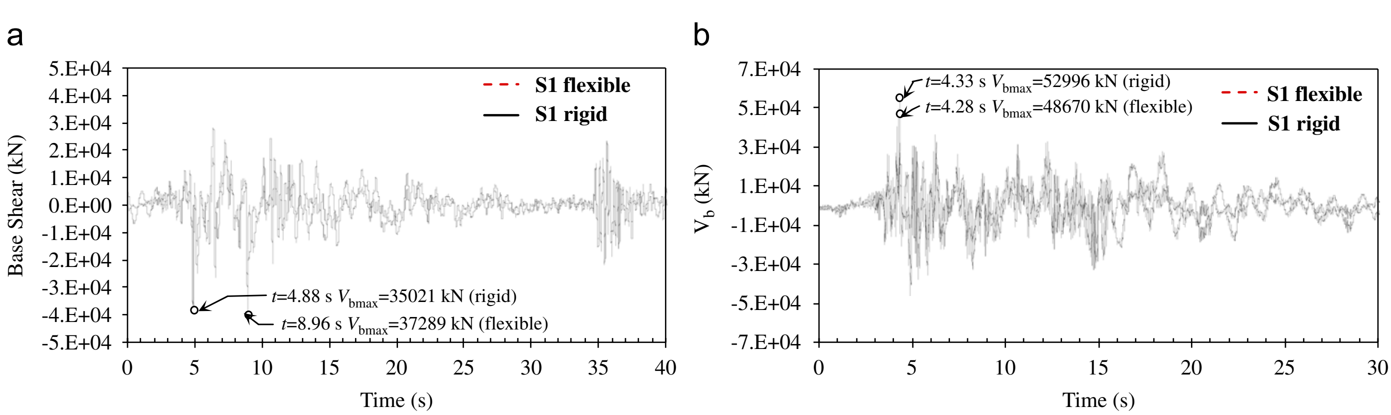

Fig. 18. Deviations of the base shear forces in time between the flexible rectangular tank and rigid tank on S6 soil type for two ground motions of (a) Kocaeli (b) Düzce Earthquakes.

Finally, it can be noted that embedment generally increase the base shear forces, however, soil conditions seemed to have a minor effect.

5. Conclusions

For the fast evaluation of the seismic behavior of the rectangular tanks, a simple procedure including fluid–structure and soil–structure interaction effects is proposed. The procedure provides not only an estimation of the base shears, overturning moments and displacements of the system but also the sloshing displacement. A computer program was also coded for the seismic analysis of rectangular tanks considering these interaction effects. Furthermore an analysis with this procedure needs less computational efforts.

Based on the above-mentioned analysis procedures and discussions on the results following conclusions can be drawn:

1. The displacement responses generally changed, when soil gets softer. For the rigid-rectangular tanks, rate of the deviations becomes larger than the flexible one. However, the changes in the displacement values are negligible for the rigid tanks. For the flexible tanks on the effect on the behavior is more pronounced and has to be considered in the design. Therefore, for the flexible tanks SSI effect should be accounted for but for the rigid tank this effect may be negligible.
2. The sloshing responses are not practically affected by embedment, wall flexibility, and SSI parameters. These effects on sloshing displacements can be ignored in the evaluation of the seismic behavior of the rectangular tanks. After all, for the flexible tank, sloshing displacement is smaller than that for the rigid tank and the embedment cause a little more increase on the sloshing displacement.
3. SSI and the flexibility of the tank walls affect the base shear response of the systems. After all, the embedment increases the base shear, but it has less effect on the increases in base shear than the other parameters. As a consequence of this study, it was seen that the rigid tank case has generally smaller base shear forces than the flexible case does although the masses of rigid tanks walls are two times larger than the masses of flexible tank.
4. It was seen that the results obtained for rectangular tank in relatively stiff soils (like S1 and S2) overestimate radiation damping in the lower-frequency range. Also, it should be noted that soil–structure interaction has less importance in such soil conditions.

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